Modelling Capital in Matching Models: Implications for Unemployment Fluctuations

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Abstract

Analysis of the the standard labor-market matching model usually focuses on labor productivity as an important source business of cycles. A shortcoming of this model is that it cannot account for observed labor market fluctuations with aggregate labor productivity as the only shock in the economy. Yet analysis of this framework disregards another potentially important source of business cycle fluctuations, namely investment-specific technical change (ISTC). In order to study the implications of ISTC for the matching model, one must first introduce capital accumulation into this model. We propose a simple extension of the search model to allow for vintage capital. We take as the defining feature of vintage capital the fact that the capital content of a machine vintage cannot be adjusted after the vintage has been introduced, i.e., after investment has taken place. For a calibrated version of our vintage capital model we find that unemployment and real wages are more volatile than in the standard search model. Whether or not ISTC significantly amplifies unemployment fluctuations depends crucially on the persistence of ISTC: the less persistent it is, the higher is unemployment volatility.

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1 Introduction

Analysis of the the standard labor-market matching model usually focuses on labor productivity as an important source business of cycles. A short-coming of this model is that it cannot account for observed labor market fluctuations with labor productivity as the only shock in the economy; see, e.g., Shimer (2005). Yet analysis of this framework disregards another potentially important source of business cycle fluctuations, namely investment-specific technical change (ISTC), e.g., as recently analyzed in Fisher (2006). In order to study the implications of ISTC for the matching model, one must first introduce capital accumulation into this model. We propose a simple extension of the search model to allow for vintage capital. We take as the defining feature of vintage capital the fact that the capital content of a machine vintage cannot be adjusted after the vintage has been introduced, i.e., after investment has taken place. For a calibrated version of our 'vintage' capital model we find that unemployment and real wages are more volatile than in the standard search model. Whether or not ISTC significantly amplifies unemployment fluctuations depends crucially on the persistence of ISTC.

We characterize the U.S. labor market using Shimer (2005)'s data set for unemployment, labor-market tightness, job finding rates, and labor productivity. We follow Greenwood, Hercowitz, and Krusell (1997) and Cummins and Violante (2002) and identify ISTC with changes in the relative price of new capital. An increasing relative productivity of the capital goods producing sector is reflected in a declining relative price of new capital. The relative price of capital, which we identify with the price of producer durable equipment and software relative to the price of nondurable consumption goods and services, has been falling since the late 1950s. A closer analysis of this relative price reveals two potential breaks in its long term trend: one in the early 1950s and one in the mid 1970s. Given the possibility of a structural break in the relative price, we detrend the data using a bandpass filter which is more flexible than the filter used in Shimer (2005). Our choice of filter affects the absolute volatility of all variables—they move less—but it does not affect the relative volatilities: unemployment is ten times as volatile as is labor productivity; the relative price of capital is about as volatile and as persistent as is labor productivity, and the relative price of capital is slightly countercyclical.

We calibrate our vintage capital model along the lines of Shimer (2005), except for the determination of the surplus share parameter and the fact that the firms own their capital. For the surplus share parameter we use the observed capital income share in output. For the cost of capital we use our observations on the relative price of capital. For the simulated vintage capital economy we find it generates significant unemployment volatility only if the capital price process is not too persistent. Furthermore, if the capital price process generates significant unemployment volatility then it also generates significant real wage volatility. In the model, real wages can be substantially more volatile than is observed for the U.S. economy.

Capital is usually introduced into the matching model assuming that there is a frictionless market where firms can rent any desired quantity of capital, e.g., Pissarides (2000). Labor productivity in a worker-firm match is then reinterpreted as output net of capital rental payments. With this procedure the matching model can then be integrated into versions of the neoclassical growth model, e.g., Andolfatto (1996) and Merz (1995). Our simple vintage model fixes the capital content of machines: there is no choice of the capital stock even at the time the investment decision is made. This means that even though we study a vintage capital economy, there is no match heterogeneity. We discuss how the assumption that the capital stock in a matched firm-worker pair is continuously adjustable limits the impact of ISTC in calibrated versions of the model.

Michelacci and Lopez-Salido (2007) modify the standard capital setup for matching models and assume that firms are locked into their capital rental decisions, but that with some probability firms get the opportunity to adjust their use of capital. The fact that firms can adjust their capital stock only infrequently introduces match heterogeneity into the economy, and Michelacci and Lopez-Salido (2007) characterize an approximate solution for the economy. We outline an extension of our simple vintage economy that allows firms to choose their capital stock at the time of the investment decision, but not during the time a match is operating. Contingent on a finite number of exogenous states, we show that an equilibrium of this economy can be completely characterized in terms of a finite number of endogenous variables associated with the exogenous states.

Other related work includes Fujita and Ramey (2005), Brügemann (2005), Kennan (2005), Eyigungor (2006), and Reiter (2007). Fujita and Ramey (2005) study the employment dynamics in response to labor productivity shocks in a capital model of vacancies ('sunk costs'). They are mainly interested in the dynamic pattern (hump-shaped response) following a shock, rather than the amplification of shocks. They also do not consider shocks to vacancy posting costs. Brügemann (2005) and Kennan (2007) consider a vintage economy with permanent productivity differences and private information about the quality of machines; Brügemann (2005) argues that private information is more important than the vintage structure in accounting for labor-market fluctuations. Reiter (2007) studies a vintage model where machine vintages have persistent productivity differences, and Eyigungor (2006) studies a vintage model with match-specific capital. Both Eyigungor (2006) and Reiter (2007) observe that for the vintage structure to matter vintage productivity shocks cannot be too persistent. Unlike for our model, it is less straightforward in these settings to use the relative price of investment to parameterize the productivity process for vintages.

The fact that real-wage fluctuations are *larger* in our model than in the model with disembodied shocks is, on one level, a step in the wrong direction, since the data reveals very weak fluctuations in the average real wage. On the other hand, to the extent that there is a mechanism forcing wages to be smoothed over the cycle—so that workers' surplus become less volatile and firms' surplus more volatile—this model will display a stronger incentive for firms to enter and exit in response to shocks.[1] We do not model such a mechanism here but merely note that some recent papers explore the possibility from a variety of perspectives; see, e.g., Hall (2005), Brügemann and Moscarini (2007), and Rudanko (2007a,b). Reiter (2007) finds that long-term wage contracting as in Rudanko (2007b) can reduce real-wage volatility in a vintage model with persistent productivity differences.

We proceed as follows. In Section 2 we describe the basic matching model with vacancy posting costs and our vintage capital version with investment costs. Section 3 describes comparative statics results for permanent shocks and persistent transitory shocks. Section 4 describes to what extent our vintage model can be reinterpreted as a standard rental capital model. Section 5 finally provides a quantitative analysis of the model; it describes the models calibration and the simulation results. Section 6 concludes.

2 A basic matching model of unemployment

In the standard matching model unemployed workers and vacant firms meet randomly, and the rates at which they meet each other depend on the relative supply of vacant firms

[1] It should be noted that the extent to which wages in *new* matches display real-wage rigidity is much less clear; see Haefke, Sonntag, and van Rens (2007) for a recent account.

and unemployed workers, i.e., labor-market tightness. We consider the implications of two alternative models of job creation. The first is the standard Mortensen-Pissarides matching model where firms pay a flow cost as long as they post a vacancy until they meet a worker. Should a matched worker and firm separate, the firm again has to pay the vacancy posting cost until it meets a new worker.[2] For our alternative setup we assume that a firm purchases a machine at a fixed cost and then puts the machine into the matching pool until it meets a worker. Once a matched machine loses its worker it can return to the matching pool without any additional cost. Our second approach treats jobs as durable capital goods that are not always fully utilized.

We choose a formulation in continuous time in order to simplify some of the derivations. It is useful to first describe the stationary economy (without aggregate shocks), because that model is simpler and yet very informative about how the model with shocks behaves. Later, we introduce aggregate fluctuations due to productivity shocks and to shocks to job creation costs.

2.1 Workers and firms

There is a measure one continuum of identical workers in the economy; the model does not consider variations in the labor force, nor in the effort or amount of time worked per worker. Workers are all the same both from the perspective of their productivity and their preferences. Workers are infinitely lived and have linear utility over consumption streams over time, which means that to the extent that there are shocks, workers are risk-neutral. There is constant (exponential) discounting. One can therefore think of a worker's present discounted utility as simply the present discounted value of wages. The (net) discount rate is denoted r.

Workers are either employed or unemployed. An employed worker earns wage income w but cannot search. Unemployed workers search for jobs. Let b denote the utility flow that a worker obtains in the non-working activity when unemployed, e.g., the monetary value of leisure plus unemployment benefits net of search costs.

A firm consists of a machine. The supply of firms is potentially infinite. Production requires one worker and one firm, and every job, i.e., every matched firm-worker pair, is

[2] The main reference is Pissarides (1985). Mortensen and Pissarides (1994) extend the model to endogenous separations. The book by Pissarides (2000) contains an excellent survey of the matching models. See also Rogerson, Shimer, and Wright (2005) for a recent survey.

equally productive at any point in time. Let the production of a firm-worker pair be denoted p. Since time is continuous in the model, production occurs every instant. We assume that the value of production for a pair always exceeds the value of not working for a worker, i.e., that $p > b$. There are potentially two costs to creating a job. In standard matching models one assumes that there is a flow cost c of keeping a vacant position open. This cost is paid until the vacant position is matched with a worker, and if the match subsequently dissolves, that cost has to paid again until a new worker is met. We consider an alternative framework where a firm has to first purchase a machine at cost q. Once the firm owns the machine it can be added at no additional cost to the vacancy pool until it is matched with a worker. Should the match dissolve for exogenous reasons, the machine can be added again to the vacancy pool at no additional cost.

2.2 A frictional labor market

A "frictional" labor market is one where a vacant firm and an idle worker cannot necessarily find each other instantaneously; therefore, resources are "wasted", since either the flow cost c is paid or the machine is not used at every moment in time when a firm is idle. Let the amount of idle firms that are vacant—that have an open position—be denoted $v(t)$ and let the number of unemployed workers be $u(t)$. Lack of coordination, partial information, and heterogeneity of vacancies and workers are all factors that explain why trading is costly in the labor market.

The rate of creation of new matches, m, is given by a Cobb-Douglas matching function, M, of the number of unemployed workers, u, searching for a job and the number of vacant positions, v:

$$m(t) = Au(t)^{\alpha} v(t)^{1-\alpha}. \tag{1}$$

The flow probability that an unemployed worker meets a firm is the total number of successful matches per worker searching:

$$\lambda_w(t) = m(t)/u(t) = A\theta(t)^{1-\alpha}, \tag{2}$$

where $\theta(t) \equiv v(t)/u(t)$ defines the so-called *tightness* of the labor market. The corresponding flow probability for a firm of meeting a worker is

$$\lambda_f(t) = m(t)/v(t) = A\theta(t)^{-\alpha}. \tag{3}$$

Existing matches separate at the rate σ. The stock of unemployed workers then evolves according to

$$\dot{u}(t) = \sigma\left[1 - u(t)\right] - \lambda_w(t)u(t), \tag{4}$$

where $\dot{u}(t)$ denotes the time derivative (change per unit of time) of $u(t)$: $\dot{u}(t) = \partial u(t)/\partial t$.

For simplicity from now on we will mainly consider *steady states*: situations in which all aggregate variables are stationary over time. Thus, $u(t)$, $v(t)$, $\lambda_w(t)$, and $\lambda_f(t)$ are all constant, even though individual workers and firms face uncertainty in their particular experiences.

2.3 Values

Denote the net present value of a matched firm J (which in general would depend on time but in a steady state does not). Letting w denote the wage paid to its worker, J must satisfy

$$rJ = p - w - \varepsilon(J - V) - \delta J, \tag{5}$$

and the return on a matched firm's value is the production flow net of wage payments, plus the expected capital loss due to separation. A match may separate for two reasons. First, at rate ε a match separates without the machine being destroyed. In this case, the machine returns to the matching pool where its capital value is V, and the capital loss to the firm is $J - V$. Second, machines are destroyed, i.e., they depreciate at the rate δ, and the capital loss to the firm is J. The total separation rate is $\sigma = \varepsilon + \delta$. Similarly, the value of a vacant firm satisfies

$$rV = -c + \lambda_f(J - V) - \delta V. \tag{6}$$

Here, there is a flow loss due to the vacancy posting cost, an expected capital gain from the probability of meeting a worker, and an expected capital loss from depreciation.

Turning to the net present value of a matched worker, W, and an unemployed worker U we similarly have

$$rW = w - \sigma(W - U), \tag{7}$$

$$rU = b + \lambda_w(W - U). \tag{8}$$

The flow return from not working b could be a monetary unemployment benefit collected by the government, a monetary benefit from working in an informal market activity, or the monetary equivalent of not working in any market activity ("the value of being at home").

2.4 Equilibrium with Nash bargaining

A firm can purchase a machine at price q. Therefore, we need to require as an equilibrium condition that

$$V = q; \tag{9}$$

otherwise, no firms would enter ($V < q$) or there would be no limit to entry ($V > q$). What this assumption means is that $v(t)$ at each point in time has to adjust so that the probability of matching with workers, λ_f, which depends on $u(t)$ and on $v(t)$, is such that firms make zero profits from entering.

We assume that wages are determined by Nash bargaining. Define the total surplus of a match by $S \equiv (J - V) + (W - U)$, i.e., the sum across the firm and the worker of the value of being in a match minus the value of not being in a match. According to Nash bargaining wages are set such that the surplus is shared between the worker and firm according to the bargaining parameter β:

$$W - U = \beta S \text{ and } J - V = (1 - \beta)S. \tag{10}$$

Summing the value equations for matched workers and firms net of the value of unemployed workers and vacant firms, and using the Nash bargaining rule, we obtain the following expression for the surplus

$$S = \frac{p + c - b}{r + \sigma + \beta\lambda_w + (1 - \beta)\lambda_f}. \tag{11}$$

From the free-entry condition, (9), and the vacancy value equation, (6), we have $-c + \lambda_f(1 - \beta)S = (r + \delta)\, q$. Therefore, we can also express the surplus as

$$S = \frac{(r + \delta)\, q + c}{(1 - \beta)\lambda_f}. \tag{12}$$

These two expressions for the surplus can be combined to yield an expression that defines equilibrium labor-market tightness

$$\frac{p + c - b}{r + \sigma + \beta\lambda_w(\theta) + (1 - \beta)\lambda_f(\theta)} = \frac{(r + \delta)\, q + c}{(1 - \beta)\lambda_f(\theta)}. \tag{13}$$

The wage that supports the Nash bargaining solution can be obtained after using the surplus sharing rule in the value equation for matched workers

$$w = \beta\left[p - (r + \delta)\, q\right] + (1 - \beta) rU. \tag{14}$$

Thus the wage is a weighted average of the net-flow return from production, where we subtract the flow cost equivalent of the machine cost, and the flow return on unemployment.

In a steady state, $\dot{u}(t) = 0$, so the evolution for unemployment as given by equation (4) becomes

$$\sigma(1 - u) = \lambda_w u. \tag{15}$$

Thus, in steady state the flow into unemployment—the separation rate in existing matches times the number of matches—must equal the flow out of unemployment—the probability for each unemployed to match times the number of unemployed.

3 Comparative statics

We now analyze how different parameters influence the endogenous variables. In particular, how does unemployment respond to changes in productivity and the cost of machines? Here, we emphasize that these are *steady-state* comparisons: we find the long-run effect of the change in the parameter. For most variables—all except $u(t)$ and $v(t)$—the influence of a permanent change in the parameter is instantaneous, because θ immediately moves to its new long-run value (see the discussion in the previous section). Of course, in the section below where some of the primitives are stochastic, their changes need not be permanent, and slightly different results apply.

In sum: if, say, we are looking at a one-percent permanent increase in productivity, p, the comparative statics analysis in this section will correctly describe the effect on θ both in the long and in the short run, whereas the effect on unemployment recorded here only pertains to how it will change in the long run. The short-run effect on unemployment of a permanent change in a parameter is straightforward to derive nevertheless: it simply involves tracing out the new dynamics implied by the linear differential equation (4) evaluated at the new permanent value for λ_w (which instantaneously adopts its new value because θ does). In particular, one sees from the differential equation that, say, an increase in θ will increase λ_w and thus increase the speed of adjustment to the new steady-state rate of unemployment.

3.1 Steady-state elasticities

We are mainly interested in how the economy responds to changes in productivity, p, and the price of capital, q, but we will also record the responses to c, and b. We compute *elasticities*, i.e., we use percentage changes and ask by how many percent θ and u will change when p, q, b, and c change by 1 percent. In Appendix A we derive the relevant expressions by employing standard comparative static differentiation of (13) and (15).

Using $\hat{x}$ to denote $d\log(x) = dx/x$, it is straightforward to derive for the economy with vacancy posting costs only, $q = \delta = 0$, that

$$\hat{\theta} = \frac{r + \sigma + \beta\lambda_w}{\alpha(r + \sigma) + \beta\lambda_w}\left[\frac{p}{p - b}\hat{p} - \frac{b}{p - b}\hat{b} - \hat{c}\right], \tag{16}$$

and for the economy with investment costs only, $c = 0$,

$$\hat{\theta} = \frac{r + \sigma + \beta\lambda_w + (1 - \beta)\lambda_f}{\alpha(r + \sigma) + \beta\lambda_w}\left[\frac{p}{p - b}\hat{p} - \frac{b}{p - b}\hat{b} - \hat{q}\right]. \tag{17}$$

In both economies the response of the unemployment rate to a change in labor-market tightness is

$$\hat{u} = -(1 - u)(1 - \alpha)\hat{\theta}. \tag{18}$$

The effect of an increase in productivity: From equation (16) we see that an increase in p of one percent leads to more than a one-percent increase in θ, since $\alpha < 1$. Comparing equations (16) and (17) we see that with the same degree of labor market turnover, this effect is larger in the economy with investment costs than in the economy with vacancy posting costs. We also see that to the extent that b is close to p, the effect can be large; in fact, as p is increased the effect is weakened as $p/(p - b)$ moves closer and closer to one. Intuitively, p increases the value of matches, and given that firms capture some of the benefits of this increase in value, there will be an increase in the number of firms per worker seeking to match. The larger the fraction of the surplus going to the firm (β small), the more vacancies and market tightness will respond to a change in labor productivity. Why is this effect larger the closer b is to p? When $(p - b) \simeq 0$, the profit from creating vacancies is small, and $\theta \simeq 0$. Hence, even a small change in p induces very large changes in θ *in percentage terms.*

Because the job finding rate λ_w equals $A\theta^{1-\alpha}$, we obtain that $\hat{\lambda}_w = (1 - \alpha)\hat{\theta}$, so the effect of p on θ is higher than that on job finding rates by a constant factor $1/(1 - \alpha)$. If we look at the effect on unemployment, note from (18) that a one percent increase in θ lowers unemployment by $(1 - u)(1 - \alpha)$ percent.[3]

[3] Note the difference between percent, which we use here, and percentage points: at a zero unemployment rate, a one-percent increase in θ decreases unemployment by $1 - \alpha$ percent, but by zero percentage points.

The effects of changing b, c, and q: We see, from (16), that b has a very similar effect to that of p but with an opposite sign. Increasing b, in particular, lowers θ significantly if b is near p, but it has very little effect on θ if b is close to zero. The effects on labor-market tightness of higher vacancy-posting costs c or higher investment costs q are negative: a one percent increase in the vacancy cost lowers the labor-market tightness. Again, comparing equations (16) and (17) we see that the effects of changes in b and q in the investment-cost model are bigger than the corresponding changes in b and c in the vacancy posting cost model.

The effects on the job finding rate of all the above changes in primitives are all $(1-\alpha)$ times the effect on $\hat{\theta}$. Similarly, the effects on unemployment are $-(1-u)(1-\alpha)$ times those on $\hat{\theta}$.

3.2 The model with aggregate risk

The economy with aggregate risk can be analyzed in almost closed form—because free entry makes vacancies adjust immediately to any shock. While unemployment is a state variable, it will only influence its own dynamics (and, residually, that of vacancies and total output), whereas all other variables will depend only on the exogenous stochastic shocks in the economy. The argument that backs this logic up proceeds by construction: specify an equilibrium of this sort and show that it satisfies all the equilibrium conditions.

Consider an economy with shocks to labor productivity and the cost of capital only. Suppose that each variable can take on only a finite number of values. The state of the economy is then characterized by the pairs (p, q), and there is a finite number of states $i = 1, \ldots, n_s$. Changes of the state are determined by a Poisson process with state-contingent arrival rates, τ_i, and state-contingent transition probabilities, $\pi_{ji} = \Pr[j|i]$. The value equations can then be rewritten as

$$rJ_i = p_i - w_i - \varepsilon (J_i - V_i) - \delta J_i + \tau_i \sum_j \pi_{ji} (J_j - J_i) \tag{19}$$

$$rV_i = -c + \lambda_f (\theta_i) (J_i - V_i) - \delta V_i + \tau_i \sum_j \pi_{ji} (V_j - V_i) \tag{20}$$

$$rW_i = w_i - \sigma (W_i - U_i) + \tau_i \sum_j \pi_i (W_j - W_i) \tag{21}$$

$$rU_i = b + \lambda_w (\theta_i) (W_i - U_i) + \tau_i \sum_j \pi_{ji} (U_j - U_i) , \tag{22}$$

Adding equations (19) and (21) and subtracting equations (20) and (22) one gets the surplus equations as

$$\left[r+\sigma+\tau_i+(1-\beta)\lambda_f(\theta_i)+\beta\lambda_w(\theta_i)\right]S_i-\tau_i\sum_j\pi_{ji}S_j=p_i+c-b \tag{23}$$

For the vacancy cost posting economy the free entry conditions are

$$V_i=0 \tag{24}$$

and for the investment cost economy they are

$$V_i=q_i. \tag{25}$$

This condition also embodies a free-exit assumption: a firm can always liquidate its capital at the price for new capital and then exit.

For each economy we can combine the free entry conditions with the vacancy value equations and obtain a second system of equations for the surplus expressions. For the vacancy posting cost economy, this system of equations is

$$c=(1-\beta)\lambda_f(\theta_i)S_i \tag{26}$$

and for the investment cost economy this system of equations is

$$(1-\beta)\lambda_f(\theta_i)S_i=(r+\delta+\tau)q_i-\tau_i\sum_j\pi_{ji}q_j \tag{27}$$

Equations (23) and (26), respectively (23) and (27), define a system of equations in the vector of state-contingent labor-market tightness.

In Appendix B we derive the elasticity of labor-market tightness with respect to productivity and vacancy posting costs for persistent, but not permanent shocks. For the vacancy posting economy these elasticities are

$$\eta_{\theta,p} = \frac{r+\sigma+\beta\lambda_w}{\alpha(r+\sigma+2\tau)+\beta\lambda_w}\frac{p}{p-b} \tag{28}$$

$$\eta_{\theta,c} = -\frac{r+\sigma+2\tau+\beta\lambda_w}{\alpha(r+\sigma+2\tau)+\beta\lambda_w}. \tag{29}$$

Note that as $\tau\to 0$, the elasticities converge to the steady-state elasticities for permanent shocks. In particular, labor-market tightness responds more strongly to a productivity increase the more persistent is the increase of labor productivity. Conversely, for relatively large values of the job-finding rate and the worker surplus share, labor-market tightness responds more strongly the less persistent is the change in vacancy-posting costs.

For the investment cost economy the elasticities of labor-market tightness with respect to productivity and investment costs are

$$\eta_{\theta,p} = \frac{r+\sigma+\beta\lambda_w+(1-\beta)\lambda_f}{\alpha(r+\sigma+2\tau)+\beta\lambda_w}\frac{p}{p-b}, \tag{30}$$

$$\eta_{\theta,q} = -\frac{r+\sigma+2\tau+\beta\lambda_w+(1-\beta)\lambda_f}{\alpha[r+\sigma+2\tau+\beta\lambda_w]+(1-\alpha)\beta\lambda_w}\left(1+\frac{2\tau}{r+\delta}\right). \tag{31}$$

Note that persistence of the investment costs now plays a much bigger role: for large values of τ, i.e., when investment costs are not persistent, the response of labor-market tightness increases linearly with τ.

4 To rent or own: A digression on the treatment of capital in matching models

We will now discuss to what extent our vintage model of capital differs from the standard treatment of capital in matching models. Pissarides (2000) provides a nice description of the standard approach. At a formal level the two approaches only differ in that firms own their capital in our vintage model, whereas firms rent their capital in the standard setting. The two approaches are equivalent in the sense that we can rewrite our 'owned' capital story as a 'rented' capital story. The two approaches differ in terms of what they assume about the ability of a firm to adjust its capital stock. Our vintage capital model assumes that firms in an existing match can never adjust their capital stock in response to a change in the environment, whereas the standard model assumes that firms can adjust their capital stock continuously. This difference has implications for how changes in the relative price of capital affect outcomes.

Pissarides (2000) describes an environment where firms that are matched with workers can rent capital in a frictionless market at the rate c_u. It is assumed that the output of a matched worker-firm pair, i.e., gross labor productivity, is a function of the per-worker capital stock k,

$$y = zf(k), \tag{32}$$

where z is exogenous aggregate technical change. Thus, independently of how wages are determined, firms choose a capital stock that maximizes the net of capital payments revenue, i.e., net labor productivity:

$$p = \max_{k} \{zf(k) - c_u k\} = zf[k(c_u, z)]. \tag{33}$$

Suppose the price of capital is a function of the aggregate state. Then the capital value expression determines the state-dependent rental rates

$$rq_i = c_{ui} - \delta q_i + \tau_i \sum_j \pi_{ji} (q_j - q_i). \tag{34}$$

Changes in the price of capital affect labor-market outcomes only through their impact on net labor productivity. Since the standard matching model treats net labor productivity as a reduced-form expression, it already incorporates all effects of ISTC. Furthermore, with a Cobb-Douglas production function, $y = zf(k)^{\alpha}$ with $0 < \alpha < 1$, net-labor productivity is proportional to gross labor productivity:

$$p = (1-\alpha) zk^{\alpha} = (1-\alpha) y. \tag{35}$$

We can rewrite our vintage-capital model so that firms rent rather than own their capital. For our vintage model we assume that a firm cannot adjust the quantity of capital: k is fixed at 1. Also, we assume that firms have to rent capital once they post a vacancy, so that the state-contingent flow cost of posting a vacancy is $c_i = c_{ui}$ and the net-return from a productive match is $p_i = y_i - c_{ui}$. Since the firm rents the capital the free-entry condition is now $V_i = 0$. This vacancy-posting model is formally equivalent to our investment-cost economy where the firm owns the capital.[4]

A calibration of the vintage model will, however, have different implications for the effects of changes in the price of capital. First, capital price changes now represent shocks to vacancy posting costs. Second, conditionally on the calibration of gross labor productivity, capital price changes represent shocks to net labor productivity.

The assumption of a fixed capital stock is a bit extreme. In Appendix C we describe an extended version of our vintage capital model that allows capital-stock adjustment for firms that have not yet been matched to workers. This version retains the flavor of a vintage capital model. It is, essentially, a simplified version of Michelacci and Lopez-Salido (2007).

[4] Hagedorn and Manovskii (2006) use a combination of the standard rental model of capital and our rental version of the vintage model. They allow for an adjustable firm capital stock, but assume that vacancies have to rent capital. They also assume that the rental price of capital is constant or that the firm's production function is Cobb-Douglas.

5 Quantitative analysis

In the previous section on comparative statics we have seen how steady states change when primitives change. In particular, we have analyzed qualitatively how a persistent productivity change influences labor-market tightness—recall that the effect is the same in the short as well as in the long run—and how it influences unemployment in the long run. However, what are the magnitudes of these effects? In order to answer this question we need to assign values to the parameters, and we will do this using "calibration": we will, to the extent possible, select parameter values based on long-run or microeconomic data. Hence, we will not necessarily select those parameters that give the best fit for the time series of vacancies and unemployment, since we restrict the parameters to match other facts.

5.1 Calibration

We first calibrate the vacancy-cost model following Shimer (2005). The only difference here is that we think of a unit of time as representing a year rather than a month. The monthly interest rate is 0.004, which corresponds to an annual interest rate of about 5 percent: $r = 0.048$. The monthly hazard rate for job separations is 0.034, which corresponds to a job separation rate $\sigma = 0.415$.[5] Thus jobs last for about two and a half years on average. Shimer estimates a monthly hazard rate of 0.45 for exit from the pool of unemployed. Thus, the job-finding rate is $\lambda_w = 7.17$. Shimer estimates the elasticity of the matching function to be $\alpha = 0.72$.[6] Given the job-finding rate and the job-separation rate, condition (15) now yields steady-state unemployment of 5.5%, which is roughly consistent with the data.

Hall (2005) argues that jobs are filled at about twice the rate that unemployed workers find jobs, i.e., that labor-market tightness is about one half: $\lambda_f = 14.34$ and $\theta = 0.5$. We follow Hall and set $\theta = 0.5$, but note that the properties of the vacancy posting economy, equations (16), (28), and (29), are independent of the worker finding rate λ_f. We therefore could equally well have normalized labor-market tightness at one as Shimer (2005) does. Conditionally on labor-market tightness, θ, the matching function elasticity, α, and the job-finding rate we determine the scale parameter of the matching function, A.

[5] Given a monthly hazard rate h, the arrival rate λ for the continuous-time Poisson process defined for a unit time period of one year is $\lambda = -12\log(1-h)$.

[6] Mortensen and Nagypal (2006) point out that this estimate of the elasticity is at the lower end of the range of plausible estimates for matching elasticities, in Petrongolo and Pissarides (2001), i.e., $[0.3, 0.5]$.

For the worker surplus share the matching literature commonly appeals to the so-called Hosios (1990) condition for efficient search. The matching frictions in the economy introduce an externality since vacant firm when making their entry decision do not take into account that variations in the vacancy rate affect the rate at which they meet unemployed workers. In an economy like the present one the Hosios condition says that firm entry is socially efficient when the surplus-sharing parameter β is equal to the elasticity parameter of the matching function, α.

The system of equilibrium conditions is homogeneous of degree one in p, b, and c. Therefore, we normalize it so that $p = 1$ in steady-state. It is common to regard b as being the monetary compensation for the unemployed. The OECD (1996) computes average "replacement rates" across countries, i.e., the ratio of benefits to average wages, and concludes that, whereas typical European replacement rates can be up to 0.70, replacement rates are at most 0.20 in the United States.[7] Shimer (2005) sets b equal to 0.4, which is even beyond this upper bound for the replacement rate since it turns out that the wage is close to one in his calibration. One reason why b should be higher than 0.2 is that it also includes the value of leisure associated with unemployment. Vacancy-posting costs are then determined residually from the free-entry condition and the vacancy value equation.

The wage share of the calibrated economy with vacancy posting costs is close to one: $w/p = 0.973$. This share of wages in production is substantially higher than what we see in the NIAs, but note that, following Pissarides (2000), it can be interpreted as the wage share in output net of capital rental payments. Recently, Hagedorn and Manovskii (2006) have followed this route in their alternative calibration of the matching model with vacancy posting costs.

[Table 1 about here]

The calibration of the economy with investment costs proceeds essentially the same as the calibration of the economy with vacancy posting costs, with one exception. In order to determine the worker surplus share the observed wage income share plays a crucial role. In Appendix D we show that with investment costs the worker surplus share implied by the

[7] In the United States, unemployment insurance replaces around 60 percent of past earnings, but in the data, unemployed workers earn much less than the average wage.

observed wage income share (in total output) and the rates of labor market turnover is

$$\beta = \frac{1}{1 + \frac{1-\omega}{\omega} \frac{r+\sigma+\lambda_w}{r+\sigma+\lambda_f} \frac{1}{1-\rho}}. \tag{36}$$

We can therefore calibrate our search model along the same lines as the calibration of the growth model. In particular, we determine the worker surplus share β based on the observed wage income share: $\omega = w/y = 2/3$.

The investment rate is the flow expenditure on new capital goods divided by the flow output

$$\phi \equiv \frac{e_f c}{(1-u)\,p}. \tag{37}$$

In the Appendix we show that the steady-state investment rate implied by the labor income share, the turnover rates in the labor market, and the depreciation rate is

$$\phi = (1-\omega)\,\frac{\sigma+\lambda_f}{r+\sigma+\lambda_f}\frac{\delta}{r+\delta}. \tag{38}$$

Shimer (2005) considers variations in gross labor productivity y as the major potential source for variations in labor-market tightness. In our setup with capital investment we consider an additional source of variations, namely changes in the relative price of capital q. Greenwood, Hercovitz, and Krusell (1997) have argued that capital-embodied technical change, as reflected in the secular decline of the relative price of capital, is a major source of growth. Recently, Fisher (2004), building on the work of Greenwood et al. (1997, 2000) and its extension in Cummins and Violante (2002), has argued that this measure of technical change is a source of employment fluctuations. We measure the relative price of capital as the price of business equipment and software relative to the price of non-durable consumption goods and services. The capital goods category seems to be about right for our purposes; it is neither too broad, such as private business fixed investment that includes non-residential structures, nor too narrow, such as information technology equipment and software.[8] The average annual depreciation rate for business equipment and software is 14 percent: $\delta = 0.14$.

5.2 A preliminary analysis of amplification

Shocks in the calibrated investment-cost model have a larger impact on labor-market tightness than in the calibrated vacancy-posting model, but the impact remains limited if the

[8] The data are from Haver: private fixed investment (equipment and software) personal consumption expenditure, prices, and values, for nondurable goods and services.

shocks are persistent. In Table 2 we display the local elasticity of labor-market tightness with respect to labor productivity and investment/vacancy costs for various degrees of persistence in the shocks.

The impact of shocks to labor productivity remains limited: in the investment cost economy the response of labor-market tightness to a labor productivity shock is only about 60 percent higher than in the vacancy posting economy, independently of the persistence of the shock. This amplification mostly reflects the fact that a one-percent increase in gross labor productivity represents a one and a half percent increase in net labor productivity. This improvement is insubstantial compared to the fact that the vacancy posting cost model falls well short of what is required to match observed volatility; see Shimer (2005).

Capital price shocks can have a substantial impact in the investment-cost model if they are not persistent. If capital prices were to change on average every half-year then the amplification of shocks would be a factor of ten: firms would take advantage of temporary good investment opportunities. On the other hand, if the price of capital changes on average only every five years, then the amplification falls to a factor of about three. Whether capital prices can generate a substantial amount of unemployment volatility thus depends crucially on the persistence properties of capital prices.

[Table 2 here]

5.3 Statistical properties of the data

We now replicate Shimer (2005)'s analysis of the U.S. economy. With the exception of the relative price of capital all data are from Shimer's website. The sample covers the time period 1951-2003, the data are quarterly. The focus of the analysis is on fluctuations at the business-cycle frequencies, and hence low-frequency movements in the data should be filtered out. We therefore detrend the log of all variables. We consider three alternative filters. First, we follow Shimer (2005) who detrends the data with an Hodrick-Prescott (HP) filter with smoothing parameter 10^5, HP(10^5), which generates a very smooth trend. The standard practice in business-cycle analysis is to use an HP filter with the smoothing parameter set to 1600, HP(1600), when applied to quarterly data. This allows for a more variable trend. For the third filter we use a band-pass filter, as in Baxter and King (1999). The band-pass (BP) filter removes a similar trend as does the HP(1600)-filter, but in addition it also eliminates a high-frequency (low-periodicity) component of the data: the filter delivers cycles with a periodicity between one and a half years and eight years.

We illustrate the properties of the three filters in Figures 1, 2, and 3, which display the trend and trend deviations of the unemployment rate, labor productivity, and the relative price of capital. The fact that the smooth trend for the HP(10^5)-filter implies a relatively larger volatility of the detrended series is clearly recognizable. It is also apparent that the volatility increase relative to the HP(1600)-filter and the BP filter is the same for the unemployment rate and for labor productivity. Figure 3 illustrates a shortcoming of the HP(10^5)-filter when there is a break in the 'true' trend line. From the graph it is apparent that there were breaks in the long-run trend of the price of capital in the early 1950s and the mid-1970s. Since the HP(10^5)-filter imposes a very smooth trend, it cannot capture these breaks. As a result, the deviations from the HP(10^5) trend become extremely persistent around the time of the break.

[Figures 1, 2, and 3 here]

In Table 3 we report the detrended variables' standard deviations and first-order autocorrelation coefficients for each filter. As we can see, using a different filter affects the volatility level of the detrended series, but it does not affect the relative volatilities. Contemporaneous correlations of the detrended series are also not much affected by the choice of filter. We therefore report only the contemporaneous correlations for the band-pass filter. We can summarize the data as follows. First, unemployment (u), labor-market tightness (θ), and job finding rates (λ_w) are all much more volatile than labor productivity (y): u is about ten times more volatile, θ is about twenty times more volatile, and λ_w is about eight times more volatile. Real wages (w) are about as volatile as labor productivity, and the price of capital is more or less volatile than labor productivity p depending on the filter. Second, labor-market tightness, the job finding rate, and real wages are positively correlated with labor productivity, i.e., they are pro-cyclical. On the other hand, the unemployment rate and the price of capital are counter-cyclical. Third, all series tend to be highly auto-correlated, the most extreme case being the relative price of capital with an HP(10^5)-filter.

[Table 3 here]

For the following simulations we assume that labor productivity and the price of capital follow a first-order Markov-process.

$$\begin{bmatrix} y_t \\ q_t \end{bmatrix} = \begin{bmatrix} \rho & 0 \\ 0 & \rho \end{bmatrix} \cdot \begin{bmatrix} y_{t-1} \\ q_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{qt} \end{bmatrix}. \tag{39}$$

We choose the autocorrelation coefficient and the standard deviations of the errors so that the standard deviations and the first-order autocorrelation coefficients of the detrended series in the simulations are close to the standard deviations and the first-order autocorrelation coefficients of the detrended actual series. As we have argued above the HP(10^5)-filter probably overstates the persistence in the price of capital series. We therefore have to choose between the HP(1600)-filter and the BP-filter. It turns out that, given the assumed Markov structure for y and q, we cannot generate enough persistence in the simulated detrended time series of q for the HP(1600)-filter. We therefore use the BP filter to compare the statistical properties of actual detrended and simulated detrended series.

5.4 Simulations

The investment-cost model with shocks to labor productivity and the price of capital goes some way towards accounting for the observed volatility of unemployment and the job finding rate. This is qualified success only, however, since the model can account for at most one half of the observed unemployment volatility, and it does so only if we make the price of capital less persistent than it likely is. We note that the amplification of capital price shocks in the model is very sensitive with respect to the persistence of these shocks. Introducing capital price shocks not only increases unemployment volatility, but it also substantially increases the volatility of real wages relative to labor productivity. Essentially, this reflects the sensitivity of real wages with respect to changes in the outside option value of workers, a feature of the model already pointed out by Shimer (2005) and Hall (2005).

The simulations are performed as follows. We calibrate the continuous-time version of the model as described in the previous section. We then construct a discrete-time approximation where a time unit represents a week: $\Delta = 1/52$. For the discretization we set the arrival rates of individual state changes to be proportional to the length of the time unit; e.g., we define the job loss rate as $\sigma\Delta$ and the time discount factor as $e^{-r\Delta}$. We derive the steady state of the discretized economy and construct a log-linear approximation around the steady state.[9] We simulate the economy and time-aggregate the weekly data to obtain quarterly observations, which are then detrended with the Baxter and King (1999) bandpass filter.

The price process for capital cannot be too persistent, since otherwise it does not have an appreciable impact on the labor market. We consider three parameterizations of the shock process (39) with ρ equalling 0.995, 0.98, and 0.96. We adjust the innovations to the shock process (39) so that the unconditional standard deviations of the BP-filtered labor productivity and price of capital from the simulations are reasonably close to the observed volatilities and so that there is a weak negative correlation between detrended labor productivity and the price of capital. We report the results from our simulations in Table 4.

In the basic Mortensen-Pissarides search model with vacancy-posting costs, labor productivity shocks can account for at most one half of a percent of the observed unemployment volatility; see Table 4.A.1. This is consistent with Shimer (2005). In the investment-cost model, labor productivity shocks alone have a bigger impact on unemployment, but the change is not dramatic: labor productivity shocks alone now account for five percent of observed unemployment volatility; see Table 4.A.2(a). This is an improvement, but five percent is still a small fraction of total unemployment volatility. Once we introduce persistent changes of the price of capital, the investment-cost model can account for more than ten percent of observed unemployment volatility; see Table 4.A.2(b). With an autocorrelation coefficient (ρ) of 0.995 for the underlying weekly observations, the price of capital for the BP-filtered and time-aggregated simulated observations is quite persistent and has an autocorrelation coefficient close to that of the observed price of capital. All variables are strongly correlated contemporaneously; see Table 4.B. Labor-market tightness, the job-finding rate, and real wages are procyclical, and the unemployment rate and the price of capital are countercyclical.

The price of capital has an even larger impact on unemployment volatility if we reduce the persistence of the Markov process. For an autocorrelation coefficient of 0.98, simulated unemployment is about one third (half) as volatile as actual unemployment; see Table 4.A.2(c&d). These parameterizations of the autocorrelation coefficient are likely at the lower

[9] We have also solved the continuous-time model with a finite number of aggregate states for labor productivity and the price of capital, not using any approximations. The statistical properties of the models are very much the same. We have chosen the log-linear approximation method for the discrete-time version since it is computationally more convenient to work directly with the Markov process (39) rather than finding its Poisson-process analogue.

bound of what can be justified by data, given that the implied autocorrelation coefficients from the simulated time-aggregated and filtered data are 0.88 and 0.85.

The price of capital becomes the major source of shocks as it becomes less persistent. This is especially apparent in the accompanying increase of real wage volatility. With the least persistent capital price series, for example, the real wage is about five times more volatile than is labor productivity. Shimer (2005) has argued that for the given parameterization of the basic Mortensen-Pissarides search model, real wages are so responsive to workers' outside options that an increase in job finding rates, due to more entry because of higher labor productivity, gets immediately translated into a wage increase. This strong wage response then reduces the benefits from entry and dampens the effects of labor productivity on job finding rates and unemployment. In our investment-cost economy this same sensitivity of wages to outside options generates substantial wage volatility in excess of that generated by volatility in labor productivity. This high wage volatility is reflected in a partial wage-labor-productivity elasticity that ranges from 1.6 to 1.8.

[Table 4 here]

6 Conclusion

We have enriched the standard Mortensen-Pissarides model of business-cycle fluctuations with a second source of aggregate fluctuations: fluctuations in capital costs, or investment-specific productivity. This second source of fluctuations is now widely held as an important source of business cycles, and it is different in nature from neutral, or disembodied, productivity shocks, especially in its implications for labor markets. In particular, a below-trend value of the price of capital induces firms to enter, and thereby make a capital gain since the shock is trend-stationary and expected to return toward trend. Thus, with low persistence, investment-specific shocks can generate significant fluctuations in firm entry and, thus, in unemployment and vacancies. Our analysis is based on a continuous-time formulation and it can be solved in closed form. We calibrate the primitive parameters to relevant microeconomic and long-run data and, in the case of investment-specific technology, to relative price data for capital. We estimate the key persistence parameter for this relative price in various ways, and we conclude tentatively that persistence is a little "too high" for calling our enterprise a full success: we generate additional volatility in labor-market variables, but not enough to close, or almost close, the gap between model and data.

A second problem with our results is that the real wage fluctuates more in our setting than when investment-specific shocks are excluded from the analysis. This is a problem since real wages appear quite smooth in the data; it is a problem also for the model without our added shock, albeit a smaller problem. Exploring this property further is an important agenda for future research. First, one needs to establish to what and why extent real wages, and especially those wages which are relevant for firm entry (such as wages for starting jobs), really do not fluctuate so much. Second, one needs to further explore mechanisms that generate smoothness in the real wage. Interesting analyses in this direction include Hall (2005), Brügemann and Moscarini (2007), Rudanko (2007a, 2007b), and Reiter (2007).

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Appendix

A Steady-state comparative statics

The total differential of the equation determining equilibrium labor-market tightness (13) is

$$\begin{aligned} & \left\{ \left[\beta\left(1-\alpha\right)\frac{\lambda_w}{\theta} - \left(1-\beta\right)\alpha\frac{\lambda_f}{\theta} \right] \left(r+\delta\right) q + \beta\left(1-\alpha\right)\frac{\lambda_w}{\theta} c \right\} d\theta \\ & + \left\{ \left[r+\sigma+\delta+\beta\lambda_w + \left(1-\beta\right)\lambda_f \right] \left(r+\delta\right) \right\} dq \\ & + \left\{ r+\sigma+\delta+\beta\lambda_w \right\} dc \\ = \; & dp - db. \end{aligned}$$

The steady-state elasticities simplify considerably for the two cases we consider. In the standard matching model with flow costs of vacancies only, $q = \delta = 0$, so we obtain

$$\hat{\theta} = \frac{r+\sigma+\beta\lambda_w}{\alpha\left(r+\sigma\right)+\beta\lambda_w} \left[\frac{p}{p-b}\hat{p} - \frac{b}{p-b}\hat{b} - \hat{c} \right].$$

In the model with sunk costs only, $c = 0$, so we obtain

$$\hat{\theta} = \frac{r+\sigma+\beta\lambda_w+\left(1-\beta\right)\lambda_f}{\alpha\left(r+\sigma\right)+\beta\lambda_w} \left[\frac{p}{p-b}\hat{p} - \frac{b}{p-b}\hat{b} - \hat{q} \right].$$

B Local approximations for the economy with aggregate risk

B.1 The economy with vacancy-posting costs

Consider an economy with two aggregate states only. Either labor productivity can be low or high, or vacancy posting costs can be low or high:

$$p_1 = p\cdot\left(1-\Delta\right) \text{ and } p_2 = p\cdot\left(1+\Delta\right), \text{ with } \Delta > 0, \tag{B.1}$$

$$c_1 = c\cdot\left(1-\Delta\right) \text{ and } c_2 = c\cdot\left(1+\Delta\right), \text{ with } \Delta > 0. \tag{B.2}$$

The equilibrium surplus expressions (23) and (26) then simplify to

$$\left[r+\sigma+\beta\lambda_w\left(\theta_i\right)+\left(1-\beta\right)\lambda_f\left(\theta_i\right)+\tau\right] S_i - \tau S_{-i} = p_i - b = \tilde{p}_i \tag{B.3}$$

$$\left(1-\beta\right)\lambda_f\left(\theta_i\right) S_i = c_i. \tag{B.4}$$

If the state changes then the economy switches from state i to state $-i$: from 1 to 2 or from 2 to 1.

For a local approximation at a point where the two states are identical ($\Delta = 0$), the equilibrium is symmetric such that θ_1 goes down by the same percentage amount as that by which θ_2 goes up. For this case, we can show explicitly how the equilibrium elasticity depends on the persistence parameter τ. Consider the total differential of (B.3) and (B.4):

$$\left[\beta(1-\alpha)\frac{\lambda_w}{\theta} - (1-\beta)\alpha\frac{\lambda_f}{\theta}\right] S d\theta_i + \left[r+\sigma+\tau+\beta\lambda_w+(1-\beta)\lambda_f\right] dS_i$$
$$-\tau dS_{-i} - d\tilde{p}_i = 0 \tag{B.5}$$
$$-\left[(1-\beta)\alpha\frac{\lambda_f}{\theta}S\right] d\theta_i + \left[(1-\beta)\lambda_f\right] dS_i = dc_i. \tag{B.6}$$

Because of symmetry, $dS_i = -dS_{-i}$ and $d\theta_i = -d\theta_{-i}$, so we can rewrite these expressions as

$$0 = \left[\beta(1-\alpha)\lambda_w - (1-\beta)\alpha\lambda_f\right]\frac{d\theta_i}{\theta}$$
$$+\left[r+\sigma+2\tau+\beta\lambda_w+(1-\beta)\lambda_f\right]\frac{dS_i}{S} - \frac{\tilde{p}}{S}\frac{d\tilde{p}_i}{\tilde{p}} \tag{B.7}$$
$$0 = -\alpha\frac{d\theta_i}{\theta} + \frac{dS_i}{S} - \frac{dc_i}{c}. \tag{B.8}$$

Start with the case where labor productivity has two states and the vacancy-posting cost is constant. With no change in the vacancy cost, expression (B.8) simply states $\hat{S}_i = \alpha\hat{\theta}_i$, which we can substitute into (B.7). After some algebra we obtain

$$\eta_{\theta,p} = \frac{r+\sigma+\beta\lambda_\omega}{\alpha(r+\sigma+2\tau)+\beta\lambda_w}\frac{p}{p-b}. \tag{B.9}$$

Now consider the case where the vacancy posting cost has two states and labor productivity is fixed. Then we can solve (B.8) for $\hat{S}_i$ and substitute the result into (B.7). After some algebra we obtain the elasticity of labor-market tightness with respect to the cost of capital:

$$\eta_{\theta,c} = -\frac{r+\sigma+2\tau+\beta\lambda_\omega}{\alpha(r+\sigma+2\tau)+\beta\lambda_w}. \tag{B.10}$$

Inspecting the result, we note that as the aggregate state becomes more persistent, i.e., τ converges to zero, the response of labor-market tightness to productivity converges to the response to a one-time permanent shock. In particular, the absolute value of the elasticity with respect to labor productivity is higher, the more persistent the shock is.

B.2 The economy with investment costs

We can perform the same exercise for the economy with investment costs. Again suppose that there are only two aggregate states. Either labor productivity can be low or high or the price of capital can be low or high:

$$q_1 = q \cdot (1 - \Delta) \text{ and } q_2 = q \cdot (1 + \Delta), \text{ with } \Delta > 0. \tag{B.11}$$

We again obtain the equilibrium surplus expression (B.3), so expression (27) simplifies to

$$(1 - \beta) \lambda_f (\theta_i) S_i - (r + \delta + \tau) q_i + \tau q_{-i} = 0. \tag{B.12}$$

Again, consider the total differential of (**??**), expression (B.5), and the total differential of (B.12):

$$-\left[(1 - \beta) \alpha \frac{\lambda_f}{\theta} S\right] d\theta_i + [(1 - \beta) \lambda_f] dS_i - (r + \delta + \tau) dq_i + \tau dq_{-i} = 0. \tag{B.13}$$

Using symmetry, $dS_i = -dS_{-i}$ and $dq_i = -dq_{-i}$, so these expressions simplify to (B.7) and to

$$0 = -\alpha \frac{d\theta_i}{\theta} + \frac{dS_i}{S} - \left[(r + \delta + 2\tau) \frac{q}{\lambda_f (1 - \beta) S}\right] \frac{dq_i}{q}. \tag{B.14}$$

Start with the case where labor productivity has two states and the price of capital is constant. With no change in the price of capital, expression (B.14) simply states $\hat{S}_i = \alpha \hat{\theta}_i$, which we can substitute into (B.7). After some algebra we arrive at

$$\eta_{\theta,p} \equiv \frac{\hat{\theta}_i}{d\Delta} = \frac{r + \sigma + (1 - \beta) \lambda_f + \beta \lambda_w}{\alpha (r + \sigma + 2\tau) + \beta \lambda_w} \frac{p}{p - b}. \tag{B.15}$$

Note that as $\tau \to 0$, i.e., the state change becomes permanent, the elasticity converges to the steady-state elasticity.

Now consider the case where the price of capital has two states and labor productivity is fixed. Then we can solve (B.14) for $\hat{S}_i$ and substitute the result into (B.7). After some algebra we obtain the elasticity of labor-market tightness with respect to the cost of capital:

$$\eta_{\theta,q} \equiv \frac{\hat{\theta}_i}{d\Delta} = -\frac{r + \sigma + 2\tau + \beta \lambda_w + (1 - \beta) \lambda_f}{\alpha [r + \sigma + 2\tau + \beta \lambda_w] + (1 - \alpha) \beta \lambda_w} \left(1 + \frac{2\tau}{r + \delta}\right) \tag{B.16}$$

Note that as $\tau \to 0$, the elasticity converges to the steady-state elasticity.

C A vintage model with intensive and extensive margins

We have discussed two extreme ways of introducing capital into the basic Mortensen-Pissarides search model. On the one hand, Pissarides (2000) assumes that a match can continuously adjust the capital stock it rents in response to exogenous changes. Thus, the aggregate capital stock adjusts simultaneously on the extensive margin (number of firms) and on the intensive margin (capital per firm). With a Cobb-Douglas specification of the firm's production function the effects of changes in the relative price of capital are completely captured by movements in labor productivity. On the other hand, in our vintage-capital model we assume that the capital stock in each firm is fixed and that the aggregate capital stock adjusts only at the extensive margin. In this model capital price changes have effects that are not completely captured by changes in labor productivity. Michelacci and Lopez-Salido (2006) consider a hybrid environment where new entrants can always choose their capital stock, and existing firms also have limited opportunities to adjust their capital stock. We now describe a version of their environment, where the equilibrium can potentially be characterized in a way that is similar to what we have done in this paper.

As in Section 3.2, assume that there is a finite number of exogenous aggregate states i and that the evolution of these states follows a Poisson process. Assume that a firm can choose the size of its capital stock, k, before it posts a vacancy, and production of a matched firm is then determined by aggregate productivity z and the capital stock: $p = zk^{\alpha}$. A vacant firm can always sell its capital stock at the current price of capital (q). Once a firm is matched with a worker it cannot adjust its capital stock. An equilibrium of this economy involves a non-degenerate distribution over firm types, i.e., firms that operate machines with different capital content. A characterization of optimal decisions remains, however, relatively straightforward since decisions—in particular, the posting of vacancies—depend only on the exogenous aggregate state of the economy. We will show this by constructing an equilibrium with this property.

Let k_{ij} denote the capital content of a machine that is matched with a worker in state j and is operating in the current state i. The output of such a match is $p_{ij} = z_i k_j^{\alpha}$. The capital value equations (19)-(22) of the vintage economy with aggregate risk are modified as follows:

$$rJ_{ij} = p_{ij} - w_{ij} - \varepsilon \left(J_{ij} - V_{ij}\right) - \delta J_{ij} + \tau_i \sum_s \pi_{si} \left(J_{sj} - J_{ij}\right) \tag{C.1}$$

$$rV_{ii} = -c + \lambda_f \left(\theta_i\right) \left(J_{ii} - V_{ii}\right) - \delta V_{ii} + \tau_i \sum_j \pi_{si} \left(V_{si} - V_{ii}\right) \tag{C.2}$$

$$rW_{ij} = w_{ij} - \sigma \left(W_{ij} - U_i\right) + \tau_i \sum_s \pi_{si} \left(W_{sj} - W_{ij}\right) \tag{C.3}$$

$$rU_i = b + \lambda_w \left(\theta_i\right) \left(W_{ii} - U_i\right) + \tau_i \sum_s \pi_{si} \left(U_s - U_i\right), \tag{C.4}$$

where $V_{ij} = V_i\left(k_j\right)$ and analogously for J_{ij} and W_{ij}. The surplus of a match is defined as before:

$$S_{ij} = J_{ij} + W_{ij} - V_{ij} - U_i. \tag{C.5}$$

There is free entry of firms, and a firm chooses its capital stock optimally:

$$0 \geq \max V_i\left(k\right) - q_i k. \tag{C.6}$$

A vacant firm can always 'reorganize,' i.e., buy or sell additional capital so that it has the optimal capital stock for the given aggregate state. Thus, if there is an exogenous separation of the match without the machine depreciating, the firm can sell its capital stock, and the value of a firm that loses its worker is simply

$$V_{ij} = q_i k_j. \tag{C.7}$$

All firms that are vacant then choose the optimal capital stock for the current state i and there are only type (i, i) firms in the vacancy pool. Therefore the capital value of unemployed workers depends only on the current state.

In the following we will assume that there is positive entry for all states, i.e., that the free-entry condition (C.6) holds as an equality. For the time being, we also assume that there is no endogenous exit, i.e., that the surplus for all existing matches is non-negative.[10]

The surplus equations can be written as

$$\begin{aligned} \left(r + \sigma + \tau_i\right) S_{ij} - \tau_i \sum_s \pi_{si} S_{si} &= z_i k_j^{\alpha} - \left(r + \tau_i\right) U_i + \tau_i \sum_s \pi_{si} U_s \\ &\quad - \left[\left(r + \delta + \tau_i\right) q_i - \tau_i \sum_s \pi_{si} q_s\right] k_j. \end{aligned} \tag{C.8}$$

[10] Michelacci and Lopez-Salide (2006) consider the possibility of endogenous match separations. Allowing for exit in our environment is straightforward.

We obtain this expression from the sum of equations (C.1) and (C.3), using the surplus definition (C.5) and the capital value equation (C.7) for a matched firm that becomes vacant. Implicit differentiation of the surplus equation for the capital stock yields

$$\begin{aligned} & (r+\sigma+\tau_i)\frac{\partial S_i(k_j)}{\partial k}-\tau_i\sum_s \pi_{si}\frac{\partial S_s(k_j)}{\partial k} \\ = \;& \alpha z_i k_j^{\alpha-1}-\left[(r+\delta+\tau_i)\,q_i-\tau_i\sum_s \pi_{si}q_s\right], \end{aligned} \tag{C.9}$$

which we can use in the free-entry condition with equation (C.7) to obtain

$$(1-\beta)\,\lambda_{fi}\frac{\partial S_i(k_i)}{\partial k}=(r+\delta+\tau_i)\,q_i-\tau_i\sum_s \pi_{si}q_s. \tag{C.10}$$

The surplus equation (C.8) and the optimality conditions (C.6) and (C.10) can be solved for the state-contingent equilibrium values of labor-market tightness θ_i and firm capital k_i.

Let μ_j denote the measure of worker-firm pairs that are matched when the aggregate state j prevails. If the current state is i, then this employment distribution and implicitly the measure of unemployed workers evolves according to

$$\dot{\mu}_j = \begin{cases} -\sigma\mu_j & \text{if } j\neq i \\ -\sigma\mu_i+\lambda_{wi}u & \end{cases} \tag{C.11}$$

$$u = -\lambda_{wi}+\sigma\sum_j \mu_j, \tag{C.12}$$

and aggregate output is

$$p=z_i\sum_j \mu_j k_j^{\alpha}. \tag{C.13}$$

D Calibration of the investment cost economy

The investment rate in the Mortensen-Pissarides model with capital is the flow expenditure on new capital goods divided by the flow output:

$$\phi\equiv\frac{e_f q}{(1-u)\,p}=\frac{\delta\,(v+1-u)\,q}{(1-u)\,p}.$$

After substituting for the steady-state vacancy rate and the unemployment rate we obtain

$$\phi=\delta\frac{\sigma+\lambda_f}{\lambda_f}\frac{q}{p}. \tag{D.1}$$

We can use the definition of the investment share in the expression for the wage income share, substituting for q/p in (14), to arrive at

$$\begin{aligned}
\omega &= \frac{w}{p} = (1-\beta)\frac{rU}{p} + \beta\left(1 - \underbrace{\phi\frac{r+\delta}{r}\frac{\lambda_f}{\sigma+\lambda_f}}_{=\bar{\phi}}\right) \quad \text{or} \\
\lambda_f \omega &= \bar{\lambda}_f r\tilde{U} + \left(\lambda_f - \bar{\lambda}_f\right)\left(1-\bar{\phi}\right) \text{ with } \bar{\lambda}_f = (1-\beta)\,\lambda_f. \qquad \text{(D.2)}
\end{aligned}$$

Combining the surplus equation with the free-entry condition and substituting for the relative investment costs from (D.1), we obtain

$$\bar{\lambda}_f\left(1 - r\tilde{U}\right) = \bar{\phi}\left(r+\sigma+\bar{\lambda}_f\right). \qquad \text{(D.3)}$$

Substituting out $r\tilde{U}$ from (D.2) and (D.3) we arrive at the steady-state investment rate

$$\phi = (1-\omega)\frac{\sigma+\lambda_f}{r+\sigma+\lambda_f}\frac{\delta}{r+\delta}. \qquad \text{(D.4)}$$

We can use the definition of the investment rate (D.1) together with (D.4) to obtain an expression for the calibrated relative investment cost:

$$\frac{q}{p} = (1-\omega)\frac{\lambda_f}{r+\sigma+\lambda_f}\frac{1}{r+\delta}. \qquad \text{(D.5)}$$

Now start with the unemployment value equation and use the surplus sharing rule, the free-entry condition, and expression (D.5) for the relative investment cost to obtain an expression for the flow value of unemployment

$$r\tilde{U} = \rho\omega + \frac{\beta}{1-\beta}\frac{\lambda_w}{\lambda_f}\bar{\phi}. \qquad \text{(D.6)}$$

Substitute this expression in equation (D.2) and after some simplifications an expression for the worker surplus share emerges:

$$\beta = \frac{1}{1+\frac{1-\omega}{\omega}\frac{r+\sigma+\lambda_w}{r+\sigma+\lambda_f}\frac{1}{1-\rho}}. \qquad \text{(D.7)}$$

Table 1. Steady-State Calibration

Variables and parameters common to both economies		
interest rate		$r = 0.048$
replacement rate		$\rho = 0.4$
scale of matching function		$A = 8.706$
matching function elasticity		$\alpha = 0.72$
worker finding rate,		$\lambda_f = 14.34$
job finding rate,		$\lambda_w = 7.17$
job separation rate, total		$\sigma = 0.42$
unemployment rate		$u = 0.055$
Other variables and parameters		
	vacancy cost	investment cost
worker surplus share	$\beta = \alpha$	$\beta = 0.699$
wage income share	$\omega = 0.986$	$\omega = 2/3$
entry cost	$c = 0.432$	$q = 1.717$
unemployment flow payment	$b = 0.394$	$b = 0.267$
job separation rate	$\varepsilon = \sigma$	$\varepsilon = 0.28$
depreciation rate	$\delta = 0$	$\delta = 0.14$

Table 2. Elasticity of Tightness with Respect to Productivity and the Price of Capital, Local Approximation

τ	0.00	0.01	0.02	0.05	0.10	0.50
average duration (in years)	∞	25.00	12.50	5.00	2.50	0.50
elasticity of θ with respect to p, $\eta_{\theta,p}$						
with vacancy posting	1.56	1.56	1.55	1.54	1.52	1.38
with capital investment	2.50	2.49	2.48	2.46	2.43	2.20
elasticity of θ with respect to c, $\eta_{\theta,c}$						
with vacancy posting	-1.02	-1.02	-1.03	-1.03	-1.03	-1.07
with capital investment	-1.83	-2.02	-2.22	-2.79	-3.75	-11.23

Table 3. The U.S. Economy

	u	θ	λ_w	w	y/n	q
	(1) HP(10^5)-filter					
Std Devs	18.99	38.17	16.42	2.29	2.00	2.46
AR(1)	0.94	0.95	0.91	0.78	0.89	0.97
	(2) HP(1600)-filter					
Std Devs	12.48	25.66	10.63	1.59	1.34	1.25
AR(1)	0.87	0.89	0.80	0.62	0.76	0.89
	(3) BP-filter					
Std Devs	12.24	25.79	10.28	1.22	1.29	1.23
AR(1)	0.91	0.92	0.92	0.91	0.90	0.93
	(4) Cross Correlations for BP-filter					
u	1.00	-0.99	-0.97	-0.55	-0.42	0.25
θ	-	1.00	0.98	0.55	0.41	-0.27
λ_w	-	-	1.00	0.52	0.35	-0.29
w	-	-	-	1.00	0.65	-0.39
y/n	-	-	-	-	1.00	-0.23

Note: The variables are as follows: u is the unemployment rate; θ is the vacancy-unemployment ratio; λ_w is the job-finding rate; w is labor compensation; y/n is labor productivity; q^G is the price of equipment and software relative to the price of NDR consumption goods and services. Except for the relative price of capital, the data was constructed by Robert Shimer. For additional detail see his webpage `http://home.uchicago.edu/~shimer/data/mmm`. The log-level of all variables is detrended. We use different filters: (1) a Hodrick-Prescott filter with smoothing parameter 10^5, (2) a Hodrick-Prescott filter with smoothing parameter 1600, and (3&4) a Baxter-King (1999) bandpass filter for periodicities between one and a half years and 8 years and a 6 year window.

Table 4. Model Simulations

A. Standard Deviations and Autocorrelations

	u	θ	λ_w	w	y/n	q
	A.1 vacancy posting, $\rho = 0.995$					
Std Devs	0.54	2.08	0.58	1.25	1.27	0.00
AR(1)	0.91	0.90	0.90	0.90	0.90	0.00
	A.2 investment cost economy					
	A.2 (a) y/n only, $\rho = 0.995$					
Std Devs	0.78	3.05	0.85	1.83	1.25	0.00
AR(1)	0.91	0.90	0.90	0.90	0.90	0.00
	A.2 (b) y/n and q, $\rho = 0.995$					
Std Devs	1.62	6.45	1.75	2.42	1.27	1.23
AR(1)	0.90	0.90	0.90	0.90	0.90	0.90
	A.2 (c) y/n and q, $\rho = 0.98$					
Std Devs	3.58	14.77	3.88	4.13	1.26	1.22
AR(1)	0.89	0.88	0.88	0.88	0.88	0.88
	A.2 (d) y/n and q, $\rho = 0.96$					
Std Devs	6.09	29.60	6.69	6.68	1.31	1.20
AR(1)	0.86	0.85	0.85	0.85	0.85	0.85

B. Contemporaneous Cross-Correlations, $\rho = 0.995$

	u	θ	λ_w	w	y/n	q
u	1.00	-0.96	-0.97	-0.91	-0.57	0.85
θ		1.00	1.00	0.93	0.58	-0.86
λ_w			1.00	0.93	0.59	-0.87
w				1.00	0.83	-0.64
y/n					1.00	-0.13

Note: All variables are as defined in the text. Statistics are calculated for a Baxter-King (1999) bandpass filter for periodicities between one and a half years and 8 years and a 6 year window. Results are for simulations of 200 samples with quarterly data, and each sample is 50 years long.

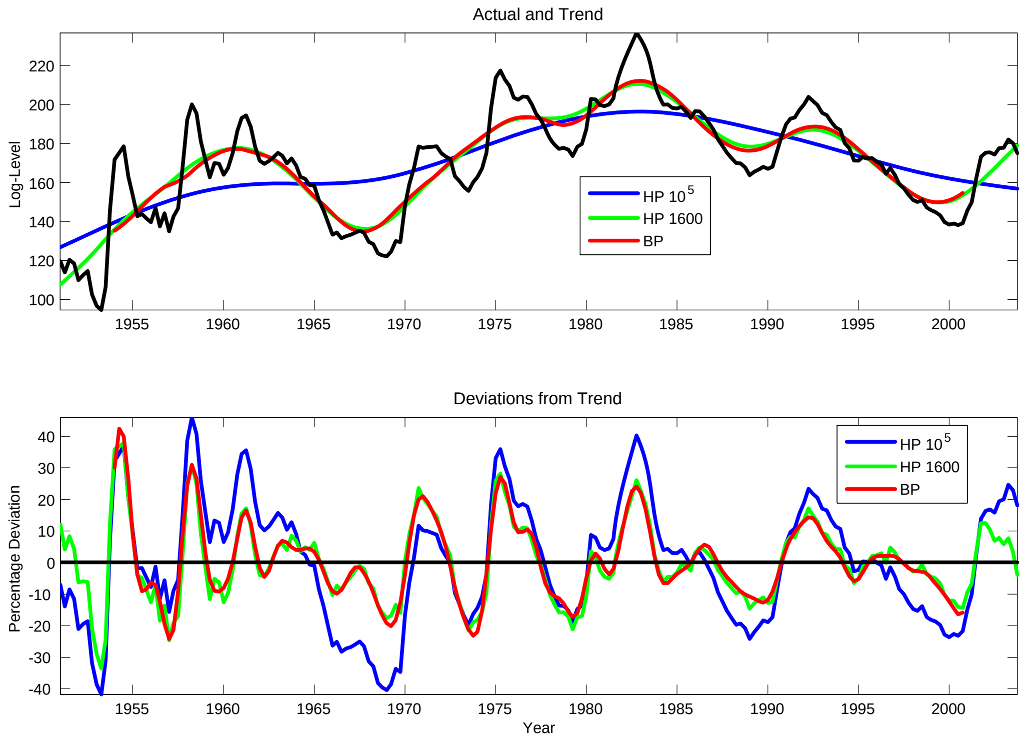

Figure 1. Unemployment Rate

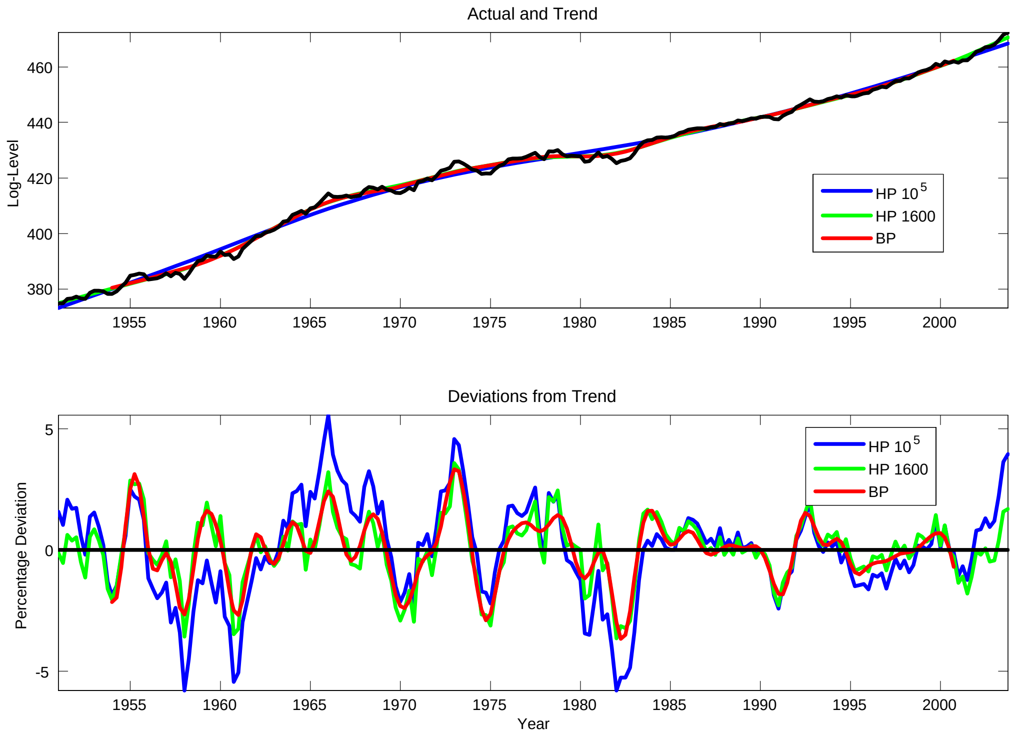

Figure 2. Labor Productivity

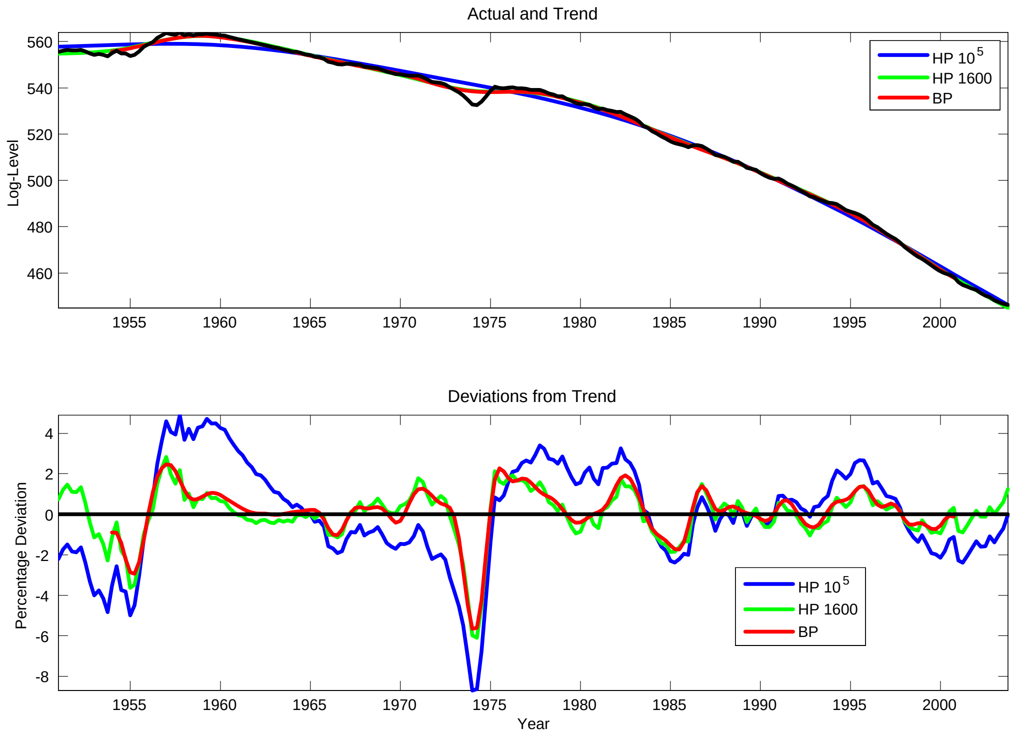

Figure 3. Price of Equipment and Software Investment relative to PCE Price of Nondurable Goods and Services